GCSE Maths – Geometry and Measures

Arc Length and Area of Sector

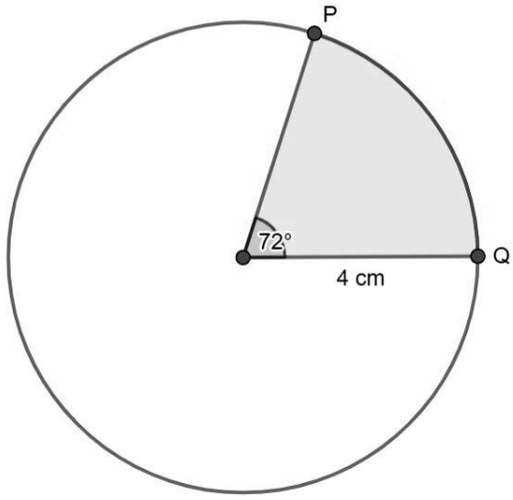
Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of arc length and sector area questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example



Calculate the major arc length of PQ to 1 decimal place.

Step 1: Work out numerical values for r and θ.

$$r = 4\ cm$$

For the major arc length, we use the angle that is greater than 180°.
Angles around a point add up to 360°:

$$\theta = 360° - 72°$$
$$\theta = 288°$$

Step 2: Substitute the values into the equation.

$$Arc\ Length = 2\pi r \times \frac{\theta}{360}$$

$$Arc\ Length = 2\pi(4) \times \frac{288}{360}$$

Step 3: Calculate the numerical value of the arc length and round the answer to the required degree of accuracy.

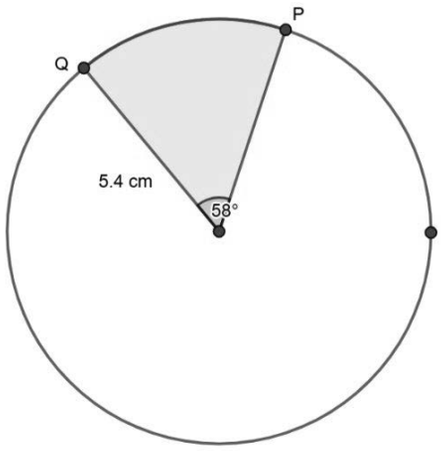
$$Arc\ Length = \left[2(4) \times \frac{288}{360}\right]\pi$$

$$Arc\ Length = 6.4\pi$$

$$Arc\ Length = 20.106\ldots cm$$

The major arc length PQ is $20.1\ cm$ to 1 decimal place.

Guided Example



Calculate the minor arc length of PQ to 1 decimal place.

Step 1: Work out numerical values for r and θ.

$r = 5.4$ cm

$\theta = 58$

Step 2: Substitute values into the equation.

$$\text{Arc Length} = 2\pi r \times \frac{\theta}{360}$$

$$= 2\pi(5.4) \times \frac{58}{360}$$

Step 3: Calculate the numerical value of the arc length and round the answer to the required degree of accuracy.

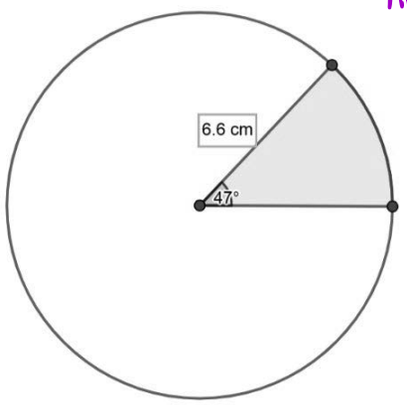
(use calculator) $= \frac{87}{50}\pi$

$= 5.466\ldots$ (6 > 5 round up)

≈ 5.5 cm (1dp)

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Calculate the minor arc length to 1 decimal place



Arc Length $= 2\pi r \times \frac{\theta}{360}$

$r = 6.6 \quad \theta = 47$

$= 2\pi(6.6) \times \frac{47}{360}$

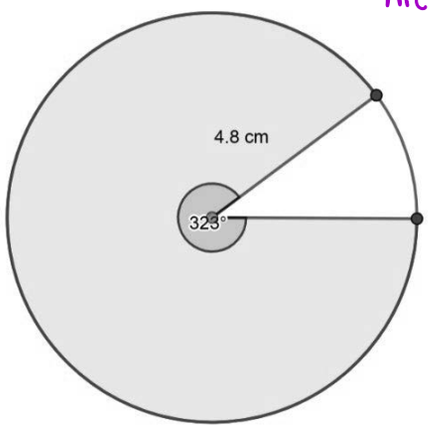
$= \frac{517}{300}\pi = 5.414\ldots$ cm

$= 5.4$ cm (1dp)

$1 < 5$ round down

2. Calculate the major arc length to 1 decimal place.

$\theta > 180$



Arc Length $= 2\pi r \times \frac{\theta}{360}$

$r = 4.8$cm $\quad \theta = 323$

$= 2\pi(4.8) \times \frac{323}{360}$

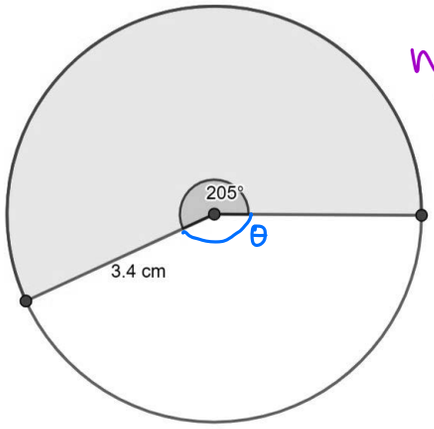
$= \frac{646}{75}\pi = 27.059\ldots$

$= 27.1$ cm (1dp)

$5 \geqslant 5$ round up

3. Calculate the minor arc length to 1 decimal place.

$\theta < 180$



Arc Length $= 2\pi r \times \frac{\theta}{360}$

We need to change θ, to calculate the minor arc length.

$\theta = 360 - 205 = 155$

$r = 3.4\text{ cm}$

$= 2\pi(3.4) \times \frac{155}{360}$

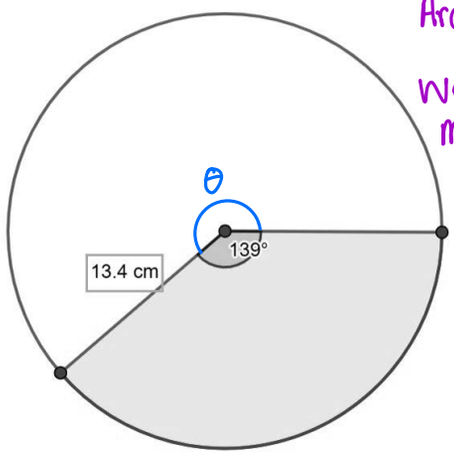
$= \frac{527}{180}\pi = 9.197\ldots$

$= 9 > 5$ round up

$= 9.2\text{ cm}$ (1dp)

4. Calculate the major arc length to 1 decimal place.

$\theta > 180$



Arc Length $= 2\pi r \times \frac{\theta}{360}$

We need to change θ, to calculate the major arc length.

$\theta = 360 - 139 = 221$

$r = 13.4\text{ cm}$

$= 2\pi(13.4) \times \frac{221}{360}$

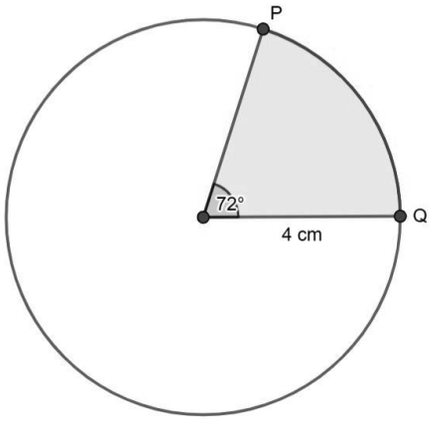
$= \frac{14807}{900}\pi = 51.686\ldots$

$= 8 > 5$ round up

$= 51.7\text{ cm}$ (1dp)

Section B

Worked Example



Calculate the area of the major sector to 1 decimal place.

Step 1: Work out numerical values for r and θ.

$$r = 4\ cm$$

For the major sector, we use the angle that is greater than 180°.

$$\theta = 360° - 72°$$

$$\theta = 288°$$

Step 2: Substitute values into the equation.

$$Area = \pi r^2 \times \frac{\theta}{360}$$

$$Area = \pi(4)^2 \times \frac{288}{360}$$

Step 3: Calculate the numerical value of the sector area and round the answer to the required degree of accuracy.

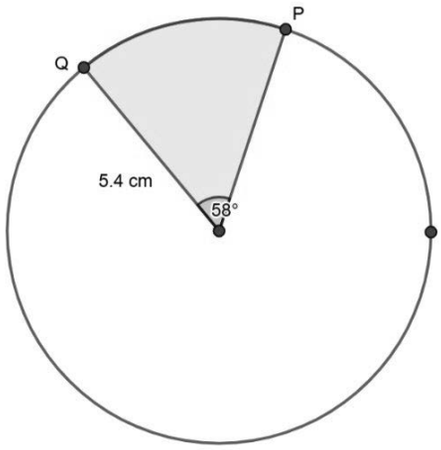
$$Area = \left[(16) \times \frac{288}{360}\right]\pi$$

$$Area = [12.8]\pi$$

$$Area = 40.21 \ldots cm^2$$

The area of the major arc sector is **40.2** cm^2 to 1 decimal place.

Guided Example



Calculate the area of the minor sector to 1 decimal place.

Step 1: Work out numerical values for r and θ.

$r = 5.4\text{cm}$

$\theta = 58$

Step 2: Substitute values into the equation.

$$\text{Area of sector} = \pi r^2 \times \frac{\theta}{360}$$

$$= \pi (5.4)^2 \times \frac{58}{360}$$

Step 3: Calculate the numerical value of the arc area in terms of π.

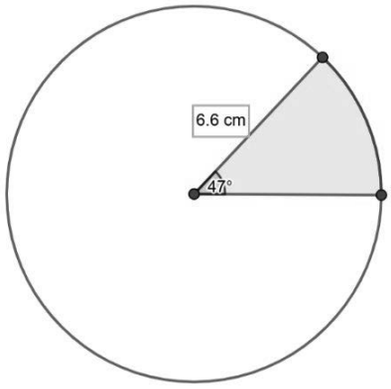
$= 14.759...$

$= 5 \geqslant 5$ round up

$= 14.8\text{ cm}^2$ (1dp)

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. Calculate the area of the minor arc sector to 1 decimal place.



Area of sector $= \pi r^2 \times \frac{\theta}{360}$

$r = 6.6 \quad \theta = 47$

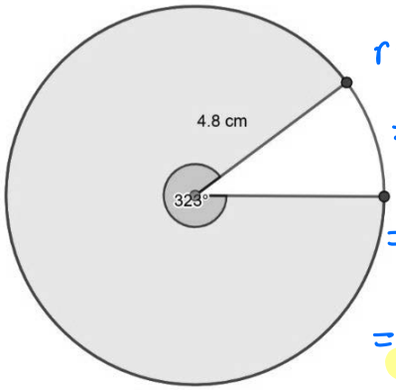
$= \pi (6.6)^2 \times \frac{47}{360}$

$= 17.866\ldots \text{ cm}^2$

$6 > 5$ round up

$= 17.9 \text{ cm}^2$ (1dp)

6. Calculate the area of the major arc sector to 1 decimal place.



$r = 4.8\text{cm} \quad \theta = 323$

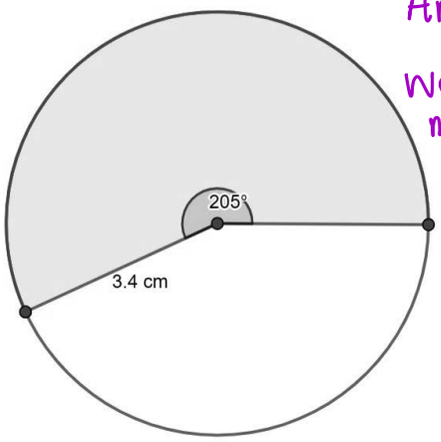
$= \pi (4.8)^2 \times \frac{323}{360}$

$= 64.943\ldots$

$4 < 5$ round down

$= 64.9 \text{cm}^2$ (1dp)

7. Calculate the area of the minor arc sector to 1 decimal place.



Area of sector $= \pi r^2 \times \frac{\theta}{360}$

We need to change θ, to calculate the minor arc length.

$\theta = 360 - 205 = 155$

$r = 3.4\text{ cm}$

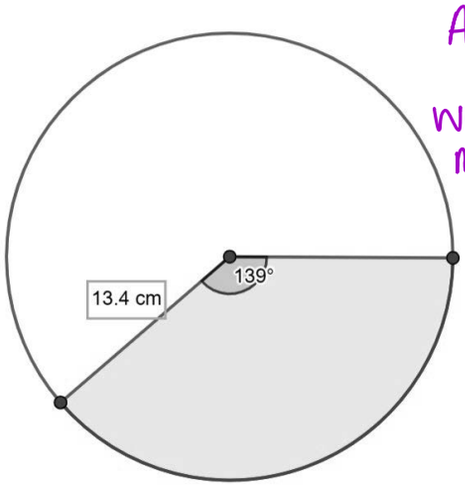
$= \pi (3.4)^2 \times \frac{155}{360}$

$= 15.636...$

$= 15.6$ (3 < 5 round down)

$= 15.6\text{ cm}^2$ (1dp)

8. Calculate the area of the major arc sector to 1 decimal place



Area of sector $= \pi r^2 \times \frac{\theta}{360}$

We need to change θ, to calculate the major arc length.

$\theta = 360 - 139 = 221$

$r = 13.4\text{ cm}$

$= \pi (13.4)^2 \times \frac{221}{360}$

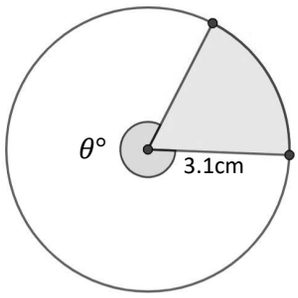
$= 346.297...$

$= 346.3$ (9 > 5 round up)

$= 346.3\text{ cm}^2$ (1dp)

Section C

Worked Example



Calculate θ to 1 decimal place when the minor arc length is 3.5 cm

Step 1: Work out numerical values for r.

$$r = 3.1\ cm$$

For minor arc length we use the angle that is less than 180°- calling this x°

Step 2: Substitute values into the equation.

$$Arc\ Length = 2\pi r \times \frac{\theta}{360}$$
$$Arc\ Length = 2\pi(3.1) \times \frac{x}{360}$$

Step 3: Calculate the numerical value of the arc length in terms of π.

$$Arc\ Length = \left[2(3.1) \times \frac{x}{360}\right]\pi$$
$$Arc\ Length = \frac{6.2x}{360}\pi$$

Step 4: Equate the arc length with the value in the question

$$3.5 = \frac{6.2x}{360}\pi$$

Step 5: Solve for the angle x

$$3.5 \div \left(\frac{6.2\pi}{360}\right) = x$$
$$64.688..° = x$$

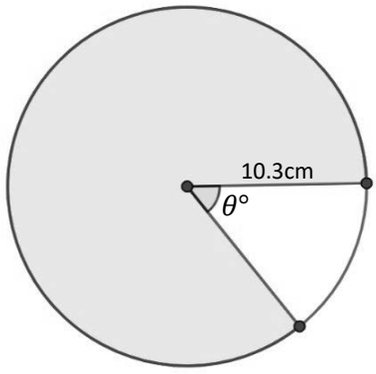
Step 6: Calculate θ

Angles around a point equal 360°

$$360 - x = \theta$$
$$360 - 64.688.. = \theta$$
$$295.31.. = \theta$$

$\theta = 295.3$ **to 1.d.p**

Guided Example



Calculate θ to 1 decimal place when the major arc length is 56.3 cm

↳ angle is greater than 180°.

Step 1: Work out numerical values for r.

$r = 10.3\text{cm}$

Step 2: Substitute values into the equation.

$$\text{Arc Length} = 2\pi r \times \frac{x}{360}$$

$$= 2\pi(10.3) \times \frac{x}{360}$$

Step 3: Calculate the numerical value of the arc length in terms of π.

$$= \frac{103x}{1800}\pi$$

Step 4: Equate the arc length with the value in the question

$$\frac{103x}{1800}\pi = 56.3$$

$$\div \frac{103}{1800}\pi$$

$$x = 313.179...$$

Step 5: Solve for x

$$\frac{103x}{1800}\pi = 56.3$$

$$\div \frac{103}{1800}\pi$$

$$x = 313.179...$$

Step 6: Calculate θ

Angles around a point = 360

$$x + \theta = 360$$

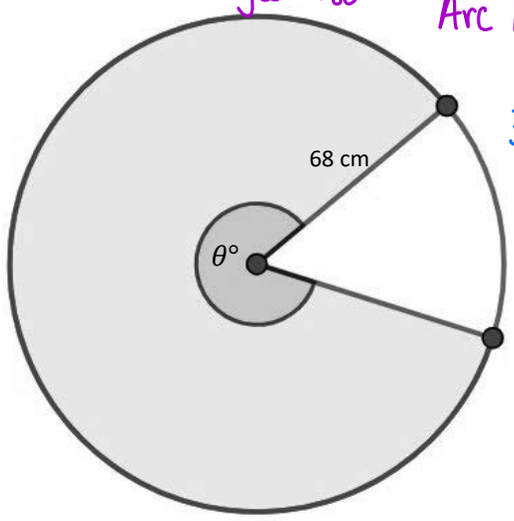
$$\theta = 360 - 313.179...$$

$$= 46.82...$$

$$= 46.8 \quad (1\text{dp})$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

9. Calculate θ to 1 decimal place when the major arc length is 341.3 cm



angle > 180

Arc Length $= 2\pi r \times \frac{\theta}{360}$

$r = 68$

$341.3 = 2\pi(68) \times \frac{\theta}{360}$

$\frac{17\theta}{45}\pi = 341.3$ cm

$\div \frac{17\pi}{45}$

$\theta = 287.57\ldots$

$= 7 > 5$ round up

$\theta = 287.6$ (1dp)

10. Calculate θ to 1 decimal place when the minor arc length is 34.2 cm

angle < 180

Arc Length $= 2\pi r \times \frac{x}{360}$

$r = 14.5$ cm

$34.2 = 2\pi(14.5) \times \frac{x}{360}$

$\frac{29x}{360}\pi = 34.2$ cm

$\div \frac{29\pi}{360}$

$x = 135.139\ldots$

$x + \theta = 360$

$\theta = 360 - 135.139\ldots$

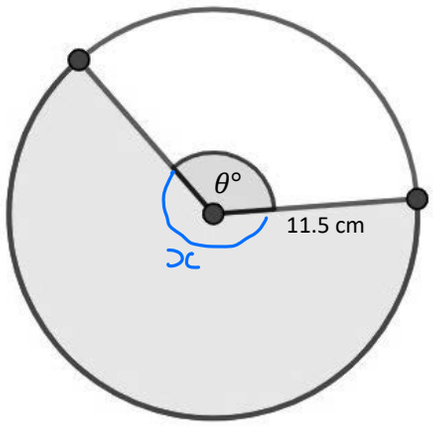
$= 224.86\ldots$

$= 6 > 5$ round up

$\theta = 224.9$ (1dp)

11. Calculate θ to 1 decimal place when the major arc length is 47.7 cm

angle > 180



Arc Length $= 2\pi r \times \frac{x}{360}$

$r = 11.5$ cm

$47.7 = 2\pi(11.5) \times \frac{x}{360}$

$\frac{23x}{360}\pi = 47.7$ cm

$\div \frac{23\pi}{360}$

$x = 237.65\ldots$

$x + \theta = 360$

$\theta = 360 - 237.65\ldots$

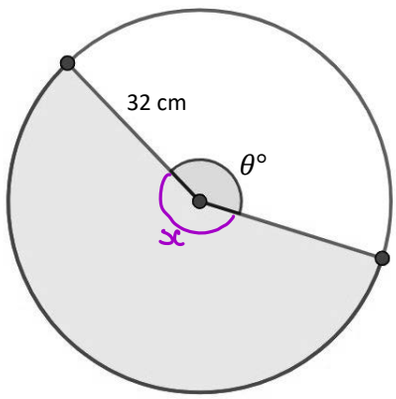
$= 122.34\ldots$

$= 4 < 5$ round down

$\theta = 122.3$ (1dp)

12. Calculate θ to 1 decimal place when the area of the major sector is 1876 cm^2

angle > 180



Area of sector $= \pi r^2 \times \frac{x}{360}$

$r = 32$ cm

$1876 = \pi(32)^2 \times \frac{x}{360}$

$\frac{128x}{45}\pi = 1876$

$\div \frac{128}{45}\pi$

$x = 209.93\ldots$

$x + \theta = 360$

$\theta = 360 - 209.93\ldots$

$= 150.06\ldots$

$6 > 5$ round up

$\theta = 150.1$ (1dp)